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Why we need more Degrees of Freedom

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Abstract

Mechanical systems encountered in biology typically have many more degrees of freedom (DOF) than the 6 DOF required to manipulate a body in space. Even the relatively rigid arthropods and crustaceans have at least 5 DOF in each limb; tentacles and human hands have many more. Robotics engineers are routinely required to choose the number of DOF in a robot in the early design stages, potentially limiting the robot's future uses. We theoretically motivate the definition of "mechanical versatility" as the ability of a mechanical system to express distinct static configurations and switch among them rapidly. Requiring versatility and assuming that the systems are power and force limited, and must furthermore resist finite energy environmental disturbances to their state, we show that such multiuse[1] mechanical systems have a lower bound on the number of DOF they require. For biomechanics, this suggests which organs and organisms will be driven to become more complex mechanically by indicating domains where higher DOF systems would intrinsically out-compete lower DOF systems.

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Nomenclature

$\mathtt{Q}$	Configuration space
Σ	Alphabet of mechanical "symbols"; a finite discrete subset of $\mathtt{Q}$
q	A configuration, $q \in \mathtt{Q}$
$\mathcal{V}$	The "mechanical versatility" of the system (defined herein)
V_{max}	Maximal velocity through configuration space
F_{max}	Maximal force exerted
W_{max}	Maximal power available
ΔE	Maximal disturbance energy

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1. Background

One of the mysteries of the natural world is the staggering mechanical complexity of organisms[2]. Why does nature select for mechanisms so much more complex than those we build? For example, why did natural selection provide many more DOF in the limbs of an animal than the 6 DOF required to arbitrarily place and orient that limb in space?

In this short paper we offer a motivating argument for why a high DOF count is an inevitable consequence of requiring "mechanical versatility" in a physically limited system. This argument has close ties to the broader question of understanding the limits and benefits of "morphological computation"[3] – the computational contribution of animal (and robot) bodies to their motions. We believe this is among the first results of this kind to be published, and hope it will motivate further research into the fundamental trade-offs inherent in embodiment of agents. The science of both biological and artificial agents could be advanced by understanding the requirements and limitations of embodied cognition[4].

We begin in section 1.1 by motivating and defining a notion of mechanical versatility using notions derived from the theory of computation.

1.1. Mechanical Versatility – a definition

To define "mechanical versatility" we rely on notions from formal language theory in computer science[5]. The intuitive essence of versatility is the presence of multiple capabilities – defined formally by the term "multi-ability" in Ferguson et al.[1] – and the further ability to rapidly switch among those capabilities. Let us indicate each such capability as symbol from a finite alphabet Σ. Versatility is thus the ability to express any string in the language Σ^* generated by this set of symbols Σ.

To obtain a simple and tractable theory of "mechanical versatility" we will take each symbol $\sigma \in \Sigma$ to be some nominal static configuration of the system. By "expressing a symbol σ" we will mean the system needs to maintain its configuration close enough to this nominal static configuration σ. Here we abuse notation, taking $\Sigma \subseteq \mathtt{Q}$ to be a discrete finite subset of the configuration space $\mathtt{Q}$ of our mechanical system. We will further and assume this configuration space to be embedded in a real space $\mathtt{Q} \subseteq \mathbb{R}^N$, which we will use to induce a norm on $\mathtt{Q}$. A symbol σ is expressed if the state $q(t)$ remains within distance ρ from σ for an interval of time of length δt, i.e. symbol σ was expressed at time t is equivalent to:

$$\forall s \in [t - \delta t, t] : \; \|q(s) - \sigma\| \leq \rho \tag{1}$$

We have thus obtained a definition of "(static configuration) mechanical versatility" in terms of the ability of the system to express arbitrary strings of (static configuration) symbols. From an information theoretic perspective, we thus suggest that a natural measure of mechanical versatility $\mathcal{V}$ is in bits – the number of bits needed to express the alphabet Σ, i.e. $\mathcal{V} := \log_2(\#\Sigma)$ where # is used to indicate set cardinality.

1.2. Adding mechanical realism to the system

Mechanical systems operate in the physical world, and comprise materials of limited strength, driven by power limited actuators. To capture some of this realism we add assumptions as follows. We assume our mechanism can only exert a force of F_{max} or less, because of limitations on its material properties and actuators. We assume the configuration space $\mathtt{Q}$ of our mechanism is compact. Finally, we also assume mechanism has a limited power budget W_{max}. The limited power budget and limited force together imply a limited maximal velocity V_{max} when changing configurations – regardless of the number of DOF being moved.

It should be noted that in both robotics and biology the limiting factor for actuator velocity is that actuator force decreases with speed. These limits derive from the underlying physics of the actuators themselves, and the non-zero dissipation arising from friction or fluid dynamics.

1.3. Environmental disturbances

The physical environment in which a system operates is never ideal. We will assume that the environment introduces arbitrary disturbances of bounded energy ΔE, which our system must resist.

To resist these disturbances and maintain the state in a neighborhood of a symbol σ, requires introducing a potential energy well that is at least ΔE deep. Doing so with a force limited to F_{max} requires a ball of radius $r := \Delta E/F_{max}$ or larger around σ to be contained within the potential well, otherwise the mechanical work done by the potential energy is insufficient to neutralize some disturbances. Thus to meet the requirement that symbols can be reliably expressed at all, our mechanism must satisfy $\rho \geq r$.

1.4. Mechanical Versatility in the face of Disturbances

Let us now examine the switching time between some pair of symbols α and β. To switch between symbols, our mechanism must escape the potential well of α and enter the ball of states which express β. As a consequence of the previous section we may conclude

$$\forall \alpha, \beta \in \Sigma : \; \|\alpha - \beta\| > 2\rho \tag{2}$$

However, because there are non-intersecting balls of radius ρ around *each and every* symbol, that is a severe underestimate. Instead, one must look to mathematical theory to estimate the density of packing of hyper-spheres[6]. As as a crude underestimate, since hyper-spheres are measurable sets with the standard Borel measure $\mu(\cdot)$, one may safely estimate an upper bound on their packing density using volumes. Denoting the "ball" $B_r(x) := \{y \in \mathbb{R}^N | \, \|x - y\| < r\}$, and exploiting the fact that volume scales with the exponent of dimension, we obtain:

$$\forall x \in \mathbb{R}^N : \; \#(\Sigma \cap B_R(x)) < \frac{\mu(B_R(0))}{\mu(B_r(0))} = \frac{\mu(B_1(0))R^N}{\mu(B_1(0))r^N} = (R/r)^N \tag{3}$$

1.5. Symbol rate limits

Let us choose a τ to indicate the maximal inter-symbol delay, i.e. our mechanism is to express $\#\Sigma$ symbols at a rate of at least one for each $\delta t + \tau$ units of time. Even if the disturbances placed us at the most convenient state within $B_\rho(\alpha)$, the transition to $B_\rho(\beta)$ requires a configuration change travel distance of at least $\|\alpha - \beta\| - 2\rho$. Thus, for a given α, if we require the ability to travel to any $\sigma \in \Sigma$, then $\|\sigma - \alpha\| < 2\rho + \tau V_{max} =: R$. Choosing R in this way gives $\Sigma \subseteq B_R(\alpha)$, leading to $\#\Sigma \leq (R/r)^N$, or $\mathcal{V} \leq N \log_2(R/r)$.

2. Main results

One immediate conclusion from this argument, is that the mechanical versatility $\mathcal{V}$ is limited to:

$$\mathcal{V} = \log_2(\#\Sigma) \leq N \log_2(R/r) = N \log_2 \left(\frac{2\rho + \tau V_{max}}{r} \right) \leq N \log_2 \left(2 + \frac{\tau F_{max} V_{max}}{\Delta E} \right) \tag{4}$$

Recognizing that $\tau F_{max} V_{max}$ is the power available for stabilizing each symbol, we define $\xi := \tau F_{max} V_{max} / \Delta E$, and identify ξ as a "safety factor" or a "signal to noise ratio" – the non-dimensional power excess available for each symbol, in units of the maximal disturbance.

We can thus conclude:

$$N \geq \frac{\mathcal{V}}{\log_2(2 + \xi)} \tag{5}$$

This leads to the following prediction: *in adverse conditions where the safety margin is low – either due to large noise, or to high symbol rate – organisms and robots will require a number of DOF that scales with the mechanical versatility in bits.*

3. An illustrative example

Let us consider a case of 1, 2, and 3 DOF systems, with parameters $F_{max}, \Delta E, V_{max}$ all set to 1, and $\tau = 2$. For all these systems, each symbol must be surrounded by a unit ball associated with it. For the 1 DOF system, only 3 unit

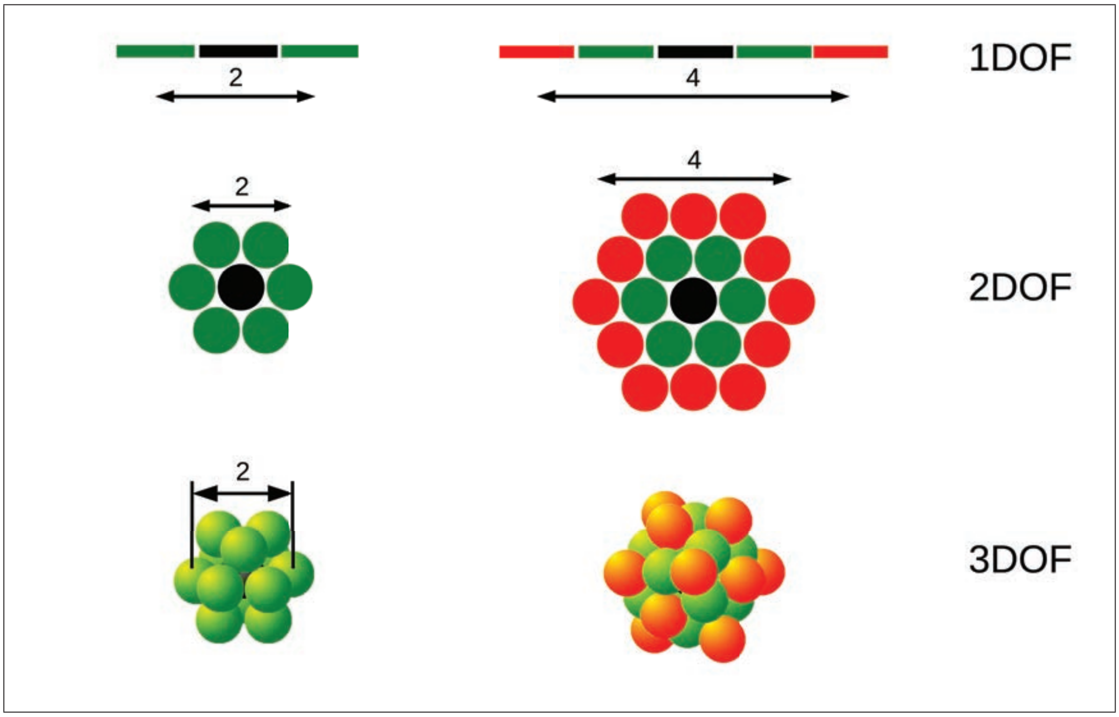

Fig. 1. An example of symbol packing in 1, 2 and 3 DOF systems. If we require symbols to be accessible within $\tau V_{max} = 2$ units ofdistance (left column) or $\tau V_{max} = 4$ units of distance (right column), the number of available symbols in the alphabet changes. Starting with a central symbol (black), there is one layer of adjacent symbols (green) in the $\tau V_{max} = 2$ case, and there is anadditional layer (red) in the $\tau V_{max} = 4$ case. For 1DOF systems, each of the layers contains 2 symbols. For 2DOF systems, layers increase in size linearly. For 3DOF systems, layers increase in size quadratically

balls (intervals) can be within τV_{max} of each other, and therefore at our desired rate $\tau = 2$, only 3 symbols at most can be expressed reliably. If we relax τ, the number of symbols will grow linearly with τ.

For a 2 DOF system, at most 7 unit balls (discs) can be within τV_{max} of each other, and therefore at our desired rate $\tau = 2$, 7 symbols at most can be expressed reliably. If we relax τ, the number of symbols will grow quadratically with τ.

Holding the number of symbols and τ constant, we must conclude that a mechanical system that needs to express more than 7 symbols must have at least 3 DOF.

4. Conclusion

The argument we present is very elementary, and includes many simplifying assumptions. We make no claim that these assumptions correctly describe the constraints placed on real world systems, but rather that they capture an essential fact – mechanical versatility requires complexity in terms of the number of degrees of freedom. While more subtle and realistic formulation of the assumptions might come closer to predicting the actual numerical bound on the number of DOF in a given collection of mechanical systems, the existence of this inherent scaling relationship whereby the number of DOF grows with the logarithm of the number of mechanical symbols seems nearly inescapable.

The core of our argument is that expressing symbols in manner robust to noise implies a packing problem for the stability basins of the symbols, and that the limitations of packing neighboring balls bound the rate of expression of such symbols. The consequence is that at a given symbol rate and noise, the number of bits per symbol expressed – the mechanical versatility – scales at most linearly with the number of DOF. Thus a high number of DOF is an essential requirement for expressing many distinct symbols rapidly and robustly.

Our astute reader may wonder why organisms would need to express many symbols in the sense we describe here. The inspiration for this notion comes from looking at locomotion and manipulation[2]. In both cases, organisms must rapidly and deftly adopt a set of contacts that transfer the required forces the ground or object being manipulated. While that problem requires only few DOF with flat ground, or spherical objects, it seems to require many distinct specialized configurations when manipulating complex objects or when small animals move through complex terrain. One may thus envision that the life of a mouse critically depends on escaping predators through the clutter of a forest floor, and requiring many distinct, complex, rapid, and deft foot placement choices. Thus, the mouse with many effective DOF in its feet out-competes the one whose feet are simpler, and allow fewer ways to interact with the substrate rapidly and reliably.

Finally, we draw the reader's attention to the fact that nowhere in this calculation did we presume an inertia for the system, nor did our bounds depend on the value of δt required to express a symbol. This means that our conclusion is truly non-dimensional and applies equally to mechanical systems at all length scales and time scales. The implication is that for systems of all scales, at high levels of environmental noise (or equivalently at low signal to noise ratios), versatility – the ability to express many distinct mechanical configurations – is Why we need more Degrees of Freedom.

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