

Bioinspired Legged Locomotion

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Locomotion as an Oscillator

1.1 Locomotion as an Oscillator

Virtually all animals, and even more so bipeds such as humans, move in a rhythmic way at moderate to high speeds of their available range of speeds. We refer to these as “rhythmic”, rather than the more mathematically strict term “periodic,” which is reserved for systems that have a precisely defined period within which motions repeat exactly. A rhythmic system is a stochastic system whose underlying deterministic part (the “drift” in the language of Stochastic Differential Equations) has an exponentially stable periodic solution. The cycles of legged locomotion, known as “strides”, typically vary from each other in duration and geometry of motion. As animals move slightly faster or slower, their limbs follow similar trajectories at slightly higher or lower rates. Even at a given stride frequency animal motions exhibit variability. At least to casual observation, it seems this variability (normalized for body size), is greater in smaller animals, in animals using more legs for propulsion, and in animals moving more slowly.

Taking the “templates and anchors” perspective of the previous section, we can rephrase this observation as a statement that the so-called “phase oscillator” is the simplest template of most moderate-speed legged locomotion. In other words, the simplest model of legged locomotion is the timely progression through a repeating sequence of body postures, which happens also to include interaction with the ground that produces propulsion. For this chapter, we will refer to this cycle as the “gait cycle”.

The phase oscillator template of locomotion can be modeled as a curve in the configuration space of the animal’s body, and a velocity associated with every point on that curve. Alternatively, it can be modeled as a periodic function of time, e.g., using a Fourier series model of the body configuration as a function of “phase”.

Under sufficiently small perturbations of the environment or body posture, animal motions recover to the gait cycle after few steps. This suggests that the slightly richer structure of an “asymptotically stable oscillator” (“oscillator” for short) applies just as universally. From a mathematical perspective, an oscillator is the differential equation that governs motion within the stability basin of a cycle, i.e., the gait cycle and all bodily states that allow for the gait to be recovered.

There is a rich mathematical literature on the structure of oscillators. If we restrict our attention in that literature to those oscillators that are structurally stable and generic, i.e., oscillators which are physically observable and whose dynamics would change only a little if properties of the body and environment change slightly, all smooth oscillators share several properties. One of the most important of these is that oscillators have a phase coordinate for the entire stability basin. This phase specializes to, and is therefore consistent

with, the phase oscillator phase on the gait cycle itself. Any perturbations of the animal away from the gait cycle will typically result in a phase shift that will persist after the animal returns to the gait cycle. Since all observables of a rhythmic system must themselves be rhythmic, we may aim to estimate the phase of an animal’s phase oscillator template (dynamical phase hereon) by observing neuromechanical quantities such as body configurations, speeds of various body parts, forces and torques, and EMG or other neuronal measurements.

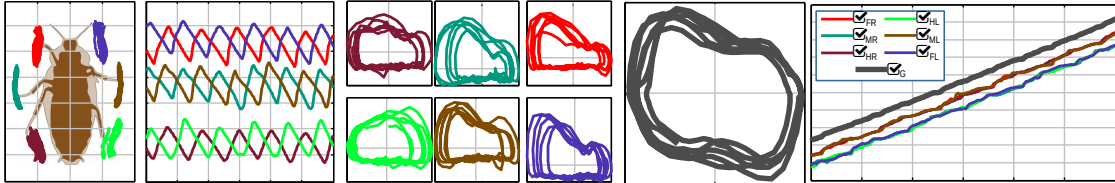


Figure 1.1: Obtaining a prediction of future motion using a phase estimate. (Left to right) Starting with cockroach foot motions in the body frame of reference (first plot), we focused on the fore-aft motions as a function of time (second plot), and computed the velocity as a function of position (middle six plots; non-dimensionalized by z-scoring). We combined these linearly to a single “foot oscillation state” with +1 coefficients for one tripod and -1 for the other. This gave rise to a state which collectively describes the phase of the gait as a whole (large oval plot). The polar angles of the plots of each leg and of the combined state follow a linear trend over multiple strides (rightmost plot). All plots shown are from the same time segment of a single experiment tracking a *Blaberus* cockroach running at moderate speed.

Once a method of phase estimation is available, predicting phase as a function of time should produce a linear trend if the phase oscillator template is compatible with the observations (see Figure 1.1). By subtracting this linear trend from the instantaneous phase estimate we can obtain the “residual phase” which can be used to identify how oscillations change under the influence of external perturbations [20].

Dynamical phase is the only dynamical variable of the phase oscillator template. The study of how that phase responds to the body and environment allows us to eliminate possible neuromechanical control architectures, e.g., by separating out responses that could be achieved only with changes to descending neural signals, and responses that could occur for solely mechanical reasons [20]. An example of such an analysis is shown in Figure 1.2.

1.2 Stride Registration as Phase Estimation

Estimating dynamical phase can also be seen as a way of representing methods of “stride registration”. Whenever we observe a rhythmically moving animal, we encounter the problem of stride registration: which samples of stride n represent “the same” state in the gait cycle as which samples of stride $n + 1$? Whenever investigators construct a notion of a gait cycle, they implicitly define such a selection method. In each such class of states which are the same in this sense, there is one distinguished representative which lies on the gait cycle itself. Because it lies on the cycle, it is a state of the phase oscillator template of that animal motion. Thus we see that any stride registration method corresponds to a choice of phase.

Typical stride registration methods in the literature include linearly interpolating once-per-stride events in time, e.g. heel strike [10, 22] or anterior extreme position of a limb [7]. Some work in robot control has attempted to parameterize a target gait using a hip-to-heel angle, or other combination of internal angles [6, 21]. By construction, these driving variables are a form of step registration as well.

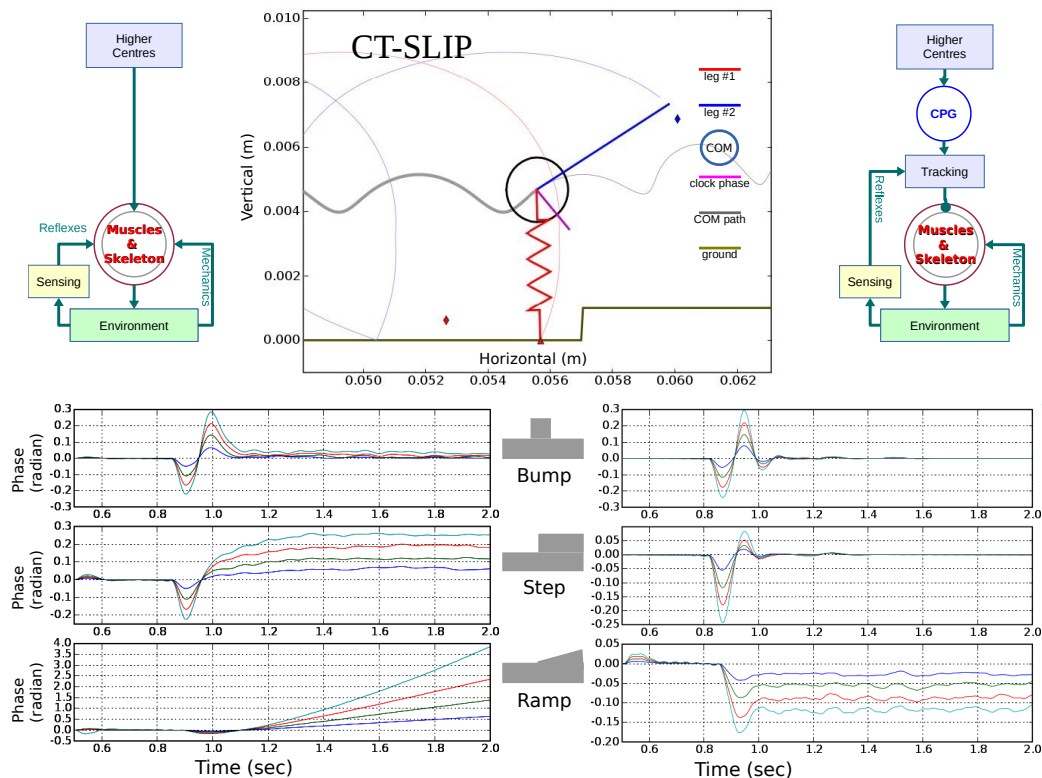


Figure 1.2: Clocked, Torqued SLIP model set to parameters of *Blaberus cockroach* running gait (upper center) with two different control architectures (upper left & right), was subjected to an assay of three perturbations (lower center) shows qualitatively different residual phase responses (plots lower left & right). In each case, the magnitude of the perturbation is varied, producing different response curves. Results show that phase alone can be used to differentiate the control architectures.

The advantage of phase based stride registration becomes clear if we assume a state independent measurement noise, and that observed motions are perturbations around a core phase oscillator template. Estimating the phase oscillator's phase and using it for binning and averaging the measurements ensures that all equal sized bins have (asymptotically) the same number of samples. Thus the bin average estimates provided are homoscedastic and standard statistical hypothesis testing tools can be used to test for treatment effects. If any other stride registration method is used the bin averages will be heteroscedastic, and require much more refined statistical techniques.

Let us compare the process of naïve stride registration and a dynamical phase-based one. For the former, we define an event detector function which has positive zero crossings when the desired event occurs, e.g. for heel-strike based stride registration we take the time and force pairs (t_i, f_i) from a force plate under the running human and renormalize to $(t_i, 1 - f_i/(mg))$. We then detect the positive crossing times $\{c_k\}$ and form the piecewise linear function of time $p(\cdot)$ such that $p(c_k) = k$. We now select a number of bins N_b and put the (multidimensional) data sample (t_i, d_i) in the bin $b_i := \lfloor N_b(p(t_i) - \lfloor p(t_i) \rfloor) \rfloor$. We estimate the period of the gait cycle τ by taking a central statistic such as the median of $\{c_{k+1} - c_k\}$. Taking a representative such

as sample average of the data in each bin in an appropriate way for the data itself, we obtain the model that at time t the gait cycle places the animal at body configuration given by the representative of the bin $\lfloor N_b(t \bmod \tau)/\tau \rfloor$.

A dynamical phase-based stride registration would consist of first training or deriving a phase estimator that gives a phase for every data sample (t_i, d_i) . Using that phase estimate instead of $p(t_i)$, i.e., by taking $b_i := \lfloor N_b(p_i - \lfloor p_i \rfloor) \rfloor$ and proceeding with the same approach to obtain bin representatives.

It should be noted that in many cases, producing the gait cycle model at a given phase does not require binning, and can instead be done by building a Fourier series model of animal properties $d(t)$ as a function of phase using a Fourier series of some order N_f :

$$x(\varphi) = \sum_{k=-N_f}^{N_f} a_k e^{i2\pi k\varphi}, \quad (1.1)$$

where

$$a_k := \int_{\text{all } t} e^{-2\pi i k t / \tau} d(t) \frac{dp}{dt}(t) dt. \quad (1.2)$$

1.3 Recovery from Perturbations

The structurally stable, generic oscillators that we use as models of locomotion share an additional property: they can be linearized exactly. The core insight dates to the late 19th century, when Gaston Floquet showed that linear time periodic (LTP) differential equations can be solved by writing their solutions as a periodic part multiplying the solutions for a linear time invariant part [Floquet, 1883]. This insight extends from LTP systems to oscillators because one may view the dynamics of the oscillator as a perturbation of the dynamics of its phase oscillator template, which is time periodic. The theory of Normal Forms [2, 11] shows that Floquet’s result does in fact extend to the entire stability basin of the oscillator.

In other words, the oscillators that appear in locomotion problems can be re-written with respect to appropriately chosen coordinates such that they are linear time invariant (LTI) systems in the new coordinates. In these linearizing coordinates, the tools of linear systems theory and control theory can be brought to bear, telling us that the long-term dynamics are governed by a single system matrix A which describes the LTI equation of motion. For a gait cycle with period τ , the matrix norm of $e^{\tau A}$ provides a bound on how quickly perturbations decay back to the unperturbed gait, with the magnitude decreasing by at least a factor of $|e^{\tau A}|$ every stride. It is important to note that in the linearizing coordinates, the results apply to both large and small perturbations.

The Floquet Normal Form provides even more detailed insight. Every perturbation to the state of the animal can be re-written in terms of a linear combination $\{\xi_k\}$ of the eigenvectors $\{v_k\}$ of A , $x(0) = \sum_k \xi_k v_k$, and will thus evolve as

$$x(t) = \sum_k e^{\lambda_k t} \xi_k v_k. \quad (1.3)$$

The “Floquet multipliers” $e^{\lambda_k \tau}$ are invariant to the choice of coordinates¹, and can therefore be computed in the original coordinates we use to obtain our measurements. Computing Floquet Multipliers is thus the method of choice for determining the stability of smooth oscillators (see Figure 1.3.).

¹This follows because the matrices involved in different coordinate representations are similar (conjugate) to each other and thus have the same eigenvalues.

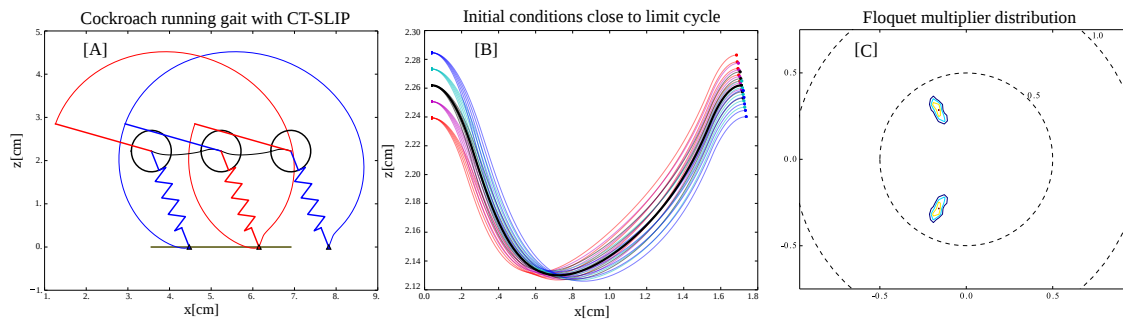


Figure 1.3: A CT-SLIP model of a running cockroach alternates between right and left foot touchdown events [A]. The center of mass bounces vertically every step, exhibiting a limit cycle ([B], heavy black line). At “apex”, with vertical velocity zero and going negative, it is convenient to define a Poincaré section. This section is 2D, consisting of height (z) and horizontal velocity (v_x) of the center of mass. An ensemble of initial conditions at apex, varying in both z and v_x ([B], colored lines) can be integrated to the next apex ([B] colored dots). Using linear regression, the affine map taking apex states to the next apex can be estimated, and its eigenvalues – the Floquet Multipliers – computed. Bootstrap analysis can further be used to get a distribution of eigenvalues and produce confidence bounds for the estimate ([C], 1000 bootstrap computations from the ensemble in [B]). For this gait the eigenvalues are a complex conjugate pair, of magnitude less than 0.5. This tells us the oscillator is very robustly stable, and perturbations decay in magnitude by better than a factor of 2 every step.

1.4 Subsystems as Coupled Oscillators

The entire argument presented above for treating a phase oscillator as a template for animal locomotion applies equally well to parts of an animal’s body. The partitioning of the animal into subsystems can be physiological, e.g., viewing the nervous system as one or more oscillators as well as viewing the musculoskeletal system as one or more oscillators. It can also follow morphology, e.g. treating each limb as an oscillator. In all such cases, one ends up with a notion of “sub-system phases” [20], and of an animal locomotion template consisting of coupled phase oscillators.

Which gait an animal is employing at any given time can be ascertained from the relative phases of the legs (see Figure 1.4).

1.5 Legged Locomotion Oscillators are Hybrid Dynamical Systems

The theory of oscillators, as described hereto, was developed for “smooth” dynamical systems – ones for which the equations of motion are at least continuously differentiable. Unfortunately, the models used for legged locomotion rarely satisfy this requirement. Typically, the equations of motion of a legged system

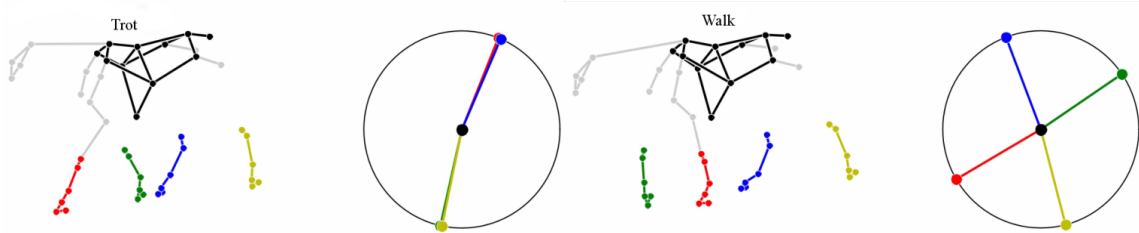


Figure 1.4: Coupled oscillator description of horse gaits. A body reference frame was fixed to the horse (black dots) and the motion of six markers on each leg was described as a phase oscillator and thereby reduced to a single phase per leg. The phases of all four legs are shown (radial lines in circles; color corresponds to animal leg) for both trotting (left) and walking (right).

depend strongly on which legs are in ground contact. Indeed, the very dimension of the system or the number of mechanical degrees of freedom may change as contact varies. For legged systems, we must extend our scope to the study of “Hybrid Dynamical Systems”. Several subtly different definitions of Hybrid Systems exist in the literature [1, 4, 5, 8, 9, 14], but all share several features: (1) the solutions of the Hybrid System are referred to as executions, rather than trajectories; (2) dynamics are defined over several domains and are smooth within each domain; (3) reset maps link domains to each other, and an execution may go through a reset map by taking its value in one domain, applying the map and using the image as the initial condition in the new domain; (4) the sets of points in each domain over which reset maps may be applied are called guards. As a concrete example which is also of interest to legged locomotion, assume we have two masses linked by a vertical spring and constrained to bounce in the vertical direction in earth gravity above level ground. While both masses are in the air, the dynamics are the smooth ballistic motion of the two masses, with the additional internal force of the connecting spring. The flight domain is four-dimensional, with two mechanical degrees of freedom (DOF). Assume further that when a mass hits the ground, it loses all kinetic energy in a plastic collision. Thus, with the lower mass on the ground, we may use a two-dimensional, one DOF model. Adding the assumption that at length 0 the spring exerts enough force to lift the top mass from the ground, we have a hybrid system with 2 domains and 4 reset maps (see Figure 1.5).

One may readily envision that with the addition of a periodic actuation force applied by the spring, the system may enter a range of persistent hopping at some constant amplitude which balances the energy lost by m colliding with the ground with the energy injected by the actuator.

While the core results of oscillator theory and Floquet theory do not apply to this system as stated, since it is not a smooth oscillator, recent results [4, 5] show that after two cycles this system becomes restricted to a 2D surface in the 4D ballistic domain, such that the motions in the stance domain and in the ballistic domain can be stitched together using a function that is smooth everywhere except the guards, and leading to dynamics that are smooth in the new coordinates. Thus, we find that once some technical complications are addressed, the long-term behavior of hybrid oscillator models that arise in legged locomotion is the same as that of the more familiar smooth oscillators.

1.6 Advanced Application: Data Driven Floquet Models

One of the strengths of the oscillator perspective on locomotion is the ability to identify properties of feasible locomotion models from observational data [16, 19, 23]. This approach has been called “Data Driven

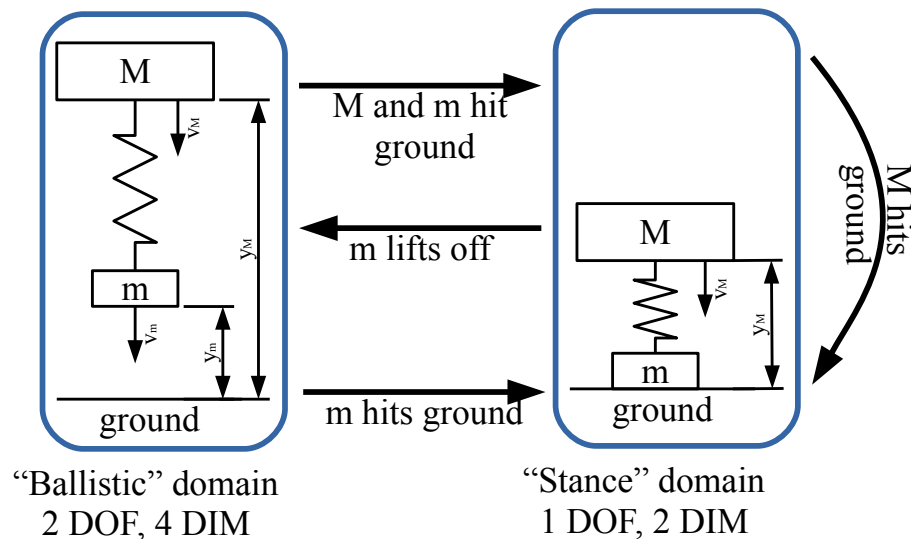


Figure 1.5: An example of a hybrid dynamical system model. A vertically bouncing pair of masses (M, m) connected by a spring can be modeled with two domains (rounded frames) and four reset maps (labeled arrows). In the ballistic flight domain, the system is 4 dimensional since the state contains position and velocity for each mass. In the Stance domain, the lower mass (m) is stationary on the ground, and the state is two dimensional, consisting only of position and velocity of one mass. Reset maps take states in which masses collide with ground to the associated Stance state, and take states in which the lower mass would detach from the ground, from stance into ballistic motion.

Floquet Analysis (DDFA)” and consists of a collection of numerical methods that attempt to reconstruct the oscillator dynamics of the putative legged locomotion oscillator directly from observational data.

One application of DDFA is the identification of plausible dimensions for template models. As described in Chapter **chapter:templates and anchors**, multiple models with varying levels of detail may exist for a given legged locomotion behavior. Viewed as an oscillator, the same behavior has a set of Floquet multipliers, the magnitudes of which define a set of decay rates. Each Floquet multiplier is associated with a Floquet mode a specific phase dependent way of the motions being offset from the limit cycle. For example, a Floquet multiplier of magnitude 0.5 would be associated with a mode that decays by a factor of two every cycle. Floquet modes evolve independently of each other, and thus any subset of modes is, in principle at least, a reduced-dimension model of the dynamics.

By the very requirement that they describe the long-term dynamics of locomotion, templates will thus comprise modes that correspond to the larger Floquet multipliers. This observation allows Floquet multipliers computed from experimental data to be sorted by magnitude and compared with the Floquet multipliers of a null (random effect) model [18]. The multipliers that cannot be accounted for by random effects may be counted, and provide an upper bound on the dimension of a template model that can reasonably be supported with those data (see Figure 1.6).

The Floquet models obtained from DDFA may be used to extend existing models of locomotion by identifying additional states that improve prediction. In the case of human running, while the SLIP model has an excellent fit to observations [12], it fails to predict stability properties, and is in fact unstable at some

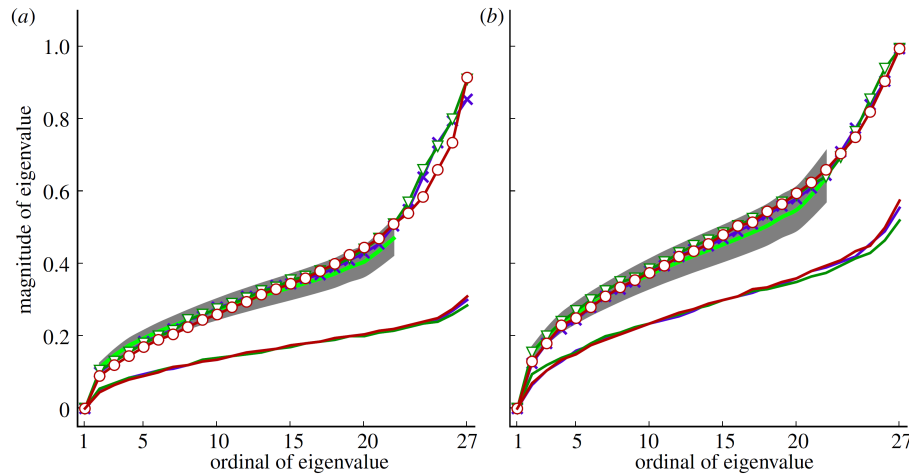


Figure 1.6: (reproduced from Figure 8 of [17]) Comparison of Floquet multiplier magnitude distributions obtained from running cockroaches. Since this analysis is done at a specific phase in the cycle, magnitudes are plotted for three different phases (0.79, 1.57, and 3.14 radians in red, green and blue). Experimental motion data is marked with markers; unmarked lines come from surrogates – randomly pairing crossings of the surface on which the Floquet multipliers are computed – and demonstrates that meaningful cycle to cycle dynamics exist. A 21-dimensional random effects model selected by the algorithm (gray confidence band with green center-line) shows the portion of the Floquet multiplier magnitudes that can be explained by random effects. In this 27-dimensional dataset, the template dynamics are therefore at most 6-dimensional.

of the range of running gait parameters humans use. In attempting to predict step-to-step running dynamics, linear feedback using an augmented SLIP model whose state consists of SLIP state variables and all SLIP parameters is less effective at predicting future states than a DDFA-derived linear model. By subjecting the DDFA model to factor analysis, five governing linear factors were obtained for a state with nearly 200 dimensions. Examination of the weights in these factors suggested that adding an ankle state could extend SLIP and give large improvements in prediction (see Figure 1.7).

1.7 Summary

At intermediate speeds, limit cycle oscillators are a useful reduced model of legged locomotion. The rich theory and tools available for analysis of oscillator dynamics provide a uniform language for expressing and understanding gaits.

Future work includes the substantial space for improvement in the numerical algorithms used for DDFA and development of algorithms that require shorter time series. Better algorithms for identifying parameters of coupled oscillator models of locomotion are needed, as most of the coupled oscillator methods from the physics literature [15] assume far weaker coupling and far lower phase noise. New directions from Koopman Theory [3] suggest a re-framing of DDFA in terms of decomposition of oscillator dynamics into Koopman modes, although numerical algorithms for accomplishing this goal are in their infancy. Finally, little to no work exists on the identification and numerical analysis of hybrid oscillators, as the theory of such oscillators is a recent addition to the field.

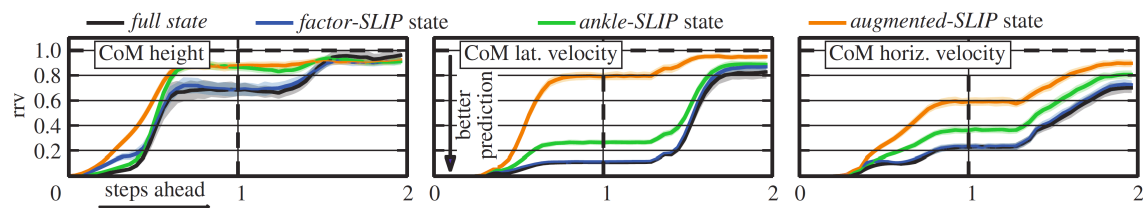


Figure 1.7: (reproduced from Figure 5, [13]) ability of various models to explain observed quantities in human running data, plotted as "relative remaining variance (rrv)": the ratio of residual variance to data variance. An rrv of 1 means no predictive ability; rrv of 0 is perfect prediction. The "full state" DDFA model, and the "factor-SLIP" model derived from it are better predictors than the "augmented SLIP" model which is itself slightly more powerful than classical Spring Loaded Inverted Pendulum models. The structure of the data driven factor-SLIP suggested adding ankle states to the system, leading to the physically meaningful ankle-SLIP model and capturing most of the potential prediction gains of the DDFA full state model.

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